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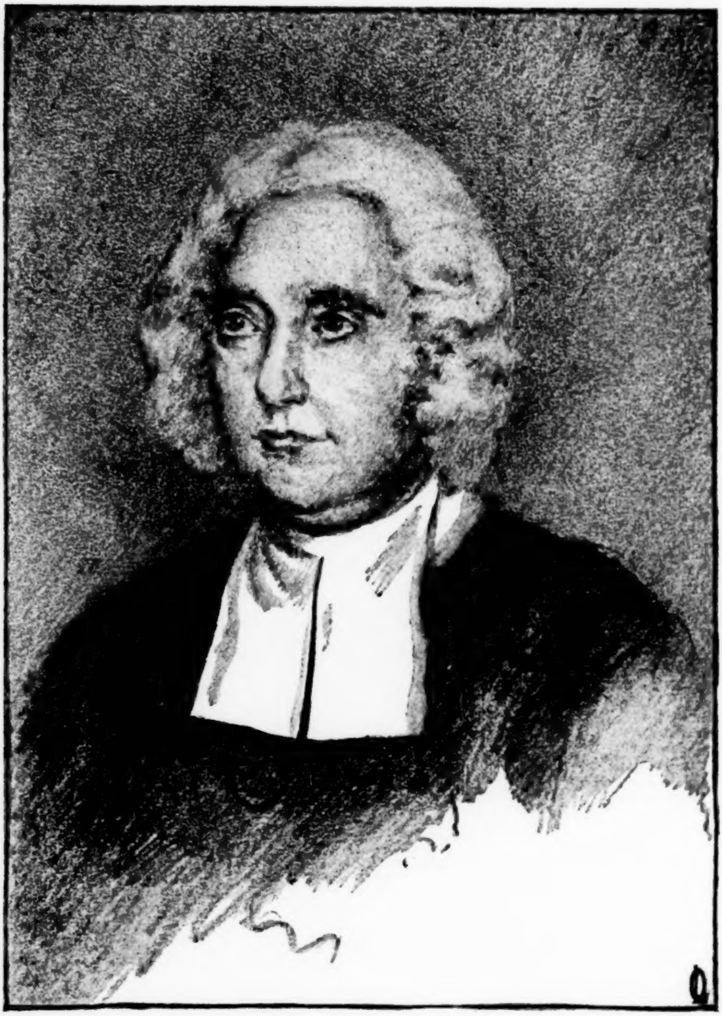

George Beckeley

1685-1753

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THE MATHEMATICS TEACHER

Volume XXVII Number 2

Edited by William David Reeve

The Training of Candidates for Teaching in Secondary Schools, Especially in Prussia

By GEORG WOLFF
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Translated by ARTHUR BEYER

AT THE LAST MEETING of the International Commission on the Teaching of Mathematics in September 1932, Professor Loria of Genoa rightly laid stress upon the fact that in the countries referred to in his report a visible conformity in the theoretical training as well as in the subsequent improvement of teachers of mathematics could be observed. However, with regard to the practical preparation for class work we may speak of a parallelism of the development only with reservation.

A rapid summary will make this clear; for there are still some great states which do not yet prepare their future teachers at all, and which, immediately after the scientific examination, send them into a school without giving them any initiation into the new activity. On the other hand, it must be acknowledged that a number of school-boards spared no pains, no power, and no expense to give the teachers of mathematics a good preparation. For every thoughtful master knows that a good piece of work cannot be accomplished without an industrious apprenticeship.

In 36% of the countries concerned, the practical training begins

during university study. Some universities confine themselves to adapting exercises in descriptive geometry or in practical analysis to the teacher's future work, others have introduced lectures in method and didactics of mathematical teaching; others have arranged pedagogical mathematical exercises; while others attend mathematical lessons with their students. One country sends the young men into the secondary school during their university study. There they are under the supervision of mathematics teachers; they have to cooperate in all the work of preparation, corrections and education; they hear continuous lessons, perhaps they try to give a lesson or two—in short they have an opportunity to decide this varied activity will be agreeable to them as a life-task. Prussia tries to solve the problem of this vital question in a similar way, by permitting students after the third term to hear lessons at a secondary school for from four to six weeks. There, if he is willing, the student takes part in the lessons as a guest, he meets the boys at games and sport, he has an opportunity by examining himself thoroughly to decide whether he is able and willing to choose the profession of a teacher.

After studying in the university the school-board sends the young man into a secondary school as "probationer." We have already said that in some states he is at once allowed to hear lessons. In others, he is attached to a teacher whose lessons he observes, and is gradually trained to hear his own lessons. That may last four months, six months or one to two years. There are also school-boards which, apart from the practical-technical initiation into teaching, demand a didactical training in methods, psychology, and practical pedagogy. In some states the probationers are obliged to fit themselves with this training. The several states conferences are occasionally arranged, some of which take place under the supervision of experienced specialist teachers and work accordingly to a strictly systematic plan.

This preparation, which is particularly professional may be concluded by an examination, the management of which is also rather different. Properly speaking it ought to be natural for countries which have no preparation also to have no examination. Nevertheless, we have two possibilities (1) no preparation and an examination (2) preparation and no examination.

The end of the practical preparation may have one of the following forms:

1. No examination
2. A lesson
3. A written and oral examination, but no lesson
4. An oral and written examination and a lesson.

According to the variety above mentioned, it is conceivable that the examination takes place either immediately after the academic studies at the same time as the academic examination, or, and this is mostly the case, after practical preparation before the class.

By this professional examination the probationer is promoted to be an "Assessor."

Also in Germany the training of candidates for teaching is not uniformly organized. The southern states of Germany, Bavaria, Wurtemberg, and Baden regard one year of training as sufficient, other states such as Bremen have no preparation of their own at all, whereas Prussia and Hamburg have a two years' course of training. The workshop of the apprentices of the profession of teaching is called "seminar," which is well to distinguish from the scientific seminar of the University.

The First Year

During the first year the seminar in Prussia is connected with a school. It is composed of probationers of the different faculties (four to six on the average). The president of this seminar is the director of the school. He is assisted by the specialist teachers. These specialists are charged with the training of the young beginers.

The training is arranged as follows:

1. The probationers hear the lessons in their own branch of instruction, but occasionally also those in the other branches.
2. In particular, they direct their attention to the lessons of their specialist teacher, who begins training by discussing his own lessons. Thus, he initiates them into the methods of instruction until they are qualified to make their appearance before the class. The success of these lessons is always seriously discussed.
3. Every week conferences take place dealing with general, historical, experimental, psychological, and practical pedagogy with reports of the members and discussions
4. Mostly every fortnight the probationers assemble in a special conference dealing with subjects like mathematics. What is

the subject matter of these special conferences? There are no official prescriptions for it. But it is desirable that the following subjects be discussed:

a. The actual mathematical curricula for the different types of schools in the country, also comparisons of these curricula with those of former times.
b. The relation of mathematics to the other branches of instruction.
c. The most important publications of the International Commission on the Teaching of Mathematics.
d. The phases of development of mathematical instruction, especially in the last fifty years.
e. All the branches of mathematics discussed from a didactical and methodical point of view. The probationer's attention is drawn to the special pedagogical difficulties in all topics such as the initiation into the negative number, the notion of parallels, the powers with fractional exponents, limits, projective geometry, etc.
f. That much value is attached to the practical participation of the probationer in exercises in surveying and in the making of mathematical models of pasteboard, wood, glass, metal.
g. Difficulties of correcting written mathematical exercises. The setting of tasks written at home or in class, their correction, the giving of marks and the correction by the pupils must be seriously discussed.
h. Modern manuals, the modern books of methods, and important articles in the reviews.

The Second Year

During the second year of the training the lessons are given for a longer time in the same class. The probationer is still under the supervision of a specialist teacher, but one sees that his lessons become more independent. Every week pedagogical conferences take place and every fortnight special conferences dealing with mathematics are held.

Particular emphasis is laid upon the continuation and deepening of the discussions about the didactical and methodical difficulties mentioned above under e. Possible subjects are as follows:

Arithmetical teaching, directed numbers, logarithms, equations,

the system of notation, series, the differential calculus, the extent of the integral calculus, the first lessons in geometry (propæ deutics) in the fourth class, fusion, perspective, the theorem of Pascal, conic sections, and the idea of relationship in geometry.

The following problems also may be discussed:

Philosophy and mathematics (logic, axioms, intuition), political economy and mathematics; mathematics for amusement, mathematics at the country-house* of the school, the mathematical teaching in foreign countries, mathematics and art, the theory of numbers, rough calculation, the slide-rule, the different solutions of equations, historical facts in the mathematical lessons.

Every month during the two years the probationer gives a trial lesson. In the first year this lesson takes place in the presence of the headmaster, the specialist teacher and the other probationers of the school.

As for the second year of training, Prussia has organised a district-seminar. The probationers of the various schools of a town or of a district are concentrated and form a great seminar, which is directed by a principal, who is assisted by the specialist teachers for the single branches of instruction. The trial-lessons now take place in the presence of the principal, of the specialist teacher, and some of the probationers.

The great advantage is the fact that a smaller number of teachers directing the specialist conferences is needed, and that for this purpose those teachers can be appointed who take a real interest in this difficult and responsible work. Another advantage is the greater uniformity that is obtained by the common conferences and trial-lessons.

For the arrangement of a district-seminar it is indispensable that a building exist which is fitted out with a good library and proper rooms for the common conferences.

Only a word need be said concerning the essays of the probationers. Toward the end of the first year of the training, he has to present the first and half a year later the second. The last one is the examination paper for the so-called "assessor examination." The subject-matter of the essays is to be in connection with the probationer's own experience in teaching. Here are some illustrative topics:

1. How I treat the transformation of planes in the fourth class.

* Landheim.

2. Initiation to the study of logarithms.
3. Nomography in the middle and upper classes.
4. Geometry of vectors in the sixth class.
5. Proof and proving in the middle classes.
6. The treatment of the text-books on the occasion of equations of the first degree with one unknown.
7. Relation between the trigonometry of planes and spaces.
8. Similarity and its relation to practical life.
9. How I treat fractional arithmetic.
10. Probability in the sixth class.
11. The fourth class course in the geometry text of Godfrey and Siddons and in German books.
12. The section of Dedekind.
13. The initiation into the differential calculus according to Perry and to German manuals.
14. The principle of permanence in the middle and upper classes.
15. Are we able to treat logistics in school?
16. Axioms in the sixth class.
17. Non-Euclidian geometry in the upper classes, an essay.
18. Tests in the middle classes.
19. How I can connect the history of mathematics and the universal and social history in the lower sixth?
20. The treatment of decimals in the first and third classes.

In conclusion let us say a few words concerning the assessor-examination. It is composed of two parts, the written and the oral examination. The examination-paper is presented about a quarter of a year before the oral examination. This oral examination comprises two trial-lessons in a forenoon, e.g. one in mathematics, the other in physics.

The probationer has to give the two examination lessons in an unknown class of an unknown school in order that the examination-board may receive as objective an impression as possible of the probationer's qualification. The subject matter of his lessons are communicated to him forty-eight hours before the lessons take place only in the presence of the members of the board of examination.

In the afternoon of the same day the examination board tries to determine by an examination of several hours to what extent the examinee has familiarized himself with the teacher's profession. He is examined on mathematical methods and didactics

within the compass of the subjects above mentioned for reports in the first and second year of training. Naturally the practical faculties of the probationer are put in the foreground, but it is also important to know the basis of his mathematical foundations. The practical mathematical instruction cannot flourish without solid scientific knowledge, however well the professional training may be organized.

Registration at the National Council Meeting

As the 1934 Annual Meeting of the National Council of Teachers of Mathematics at Cleveland, Ohio, approaches, the secretary is very anxious that the members, guests and visitors register properly. Four different colored registration cards are used to classify our registration.

1. *Visitors* and *guests*, that is *non-members* register on green cards.
2. *Members* who are attending their *first* annual meeting register on white cards.
3. *Members* who have attended more than one meeting but less than half of the annual meetings register on a red card.
4. *Members* who have attended *half or more* of the annual meetings register on a blue card, indicating thereby their "true blue" loyalty to the organization. A member does not continue in the blue class unless he maintains his record of attending at least half of the annual meetings. If he does not measure up he slips back into the red class.

The following list indicates when and where our annual meetings have been held:

1920—Cleveland	1925—Cincinnati	1930—Atlantic City
1921—Atlantic City	1926—Washington	1931—Detroit
1922—Chicago	1927—Dallas	1932—Washington
1923—Cleveland	1928—Boston	1933—Minneapolis
1924—Chicago	1929—Cleveland	1934—Cleveland

Before you go to Cleveland will you decide which of the annual meetings you have attended and thus facilitate registration by asking for the proper colored registration card. Persons attending any sessions of *The Council* should register so that the secretary will have his records complete.

EDWIN W. SCHREIBER, *Secretary*

Classifying College Students on the Basis of Their Grades in Mathematics

(SECOND REPORT)

By S. Helen Taylor

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More than two years ago we reported in the Mathematics Teacher on the progress of an experiment which was then going into its fifth year at the University of Illinois. At the time of the writing of that article, the future policy of the Department of Mathematics in regard to sectioning on the basis of previous grades in mathematics had not yet been determined. Many questions came to us as to the policy adopted and for that reason a more complete report was deemed advisable. No resumé of the procedure can be given here as the report in the November, 1931 Mathematics Teacher, gave considerable detail. The results and conclusions are based on four years' work with sixty-six sections of analytic geometry under fifty-five instructors, only eleven instructors repeating in the analytics sections in the four years. Complete statistical data are given for more than fourteen hundred students with incomplete data on two hundred others. The policy of classification as adopted two years ago is now in force, so that the period of time during which such sectioning has been used is seven years.

The Department of Mathematics met together in December, 1930, for the purpose of considering certain recommendations concerning this plan of sectioning. The following paragraphs are quoted from a letter sent out before this meeting by Dr. Bailey, Chairman of Freshman Mathematics. After a discussion of the matter, the recommendations were adopted as a departmental policy by a majority vote.

On the basis of this experiment and the experience of the Department, I make the following recommendations regarding the future practice in both the four and five hour analytics courses:

1. That wherever possible three sections of the same sort (e.g., three engineering sections) be scheduled at the same hour; that where this is not possible two sections of the same sort be scheduled at the same hour. It is probable that there may need to be some single sections remaining.

2. That students registering at hours where there are groups of two or groups of

three be put into a high or a low section in the case of a group of two or into a low, middle, or high section in the case of the group of three.

3. That this sectioning be made on the basis of the grades made in college algebra and trigonometry.

4. That the Iowa tests be not given, unless it be once every three or four years to check up on the method.

5. That at the end of six weeks, a common quiz be given to both or all the sections at each hour; that on the basis of the work done and the results of this quiz any shifts which seem desirable to the instructors be made, where feasible.

6. That each instructor give his own final examination, except that instructors having sections at the same hour may give a common examination to their sections if they so desire.

Summary

The results and conclusions from the experiment are stated briefly.

1. As measured by the Iowa tests, students at the University of Illinois who have had five hours of college mathematics show a twenty-five percent increase in aptitude and a thirty percent increase in training over students entering college courses in mathematics.

2. The new type test in analytics constructed and used as an end test in conjunction with the usual old type test in analytics correlated satisfactorily with the old type test. The coefficient of correlation varied between $.69 \pm .02$ and $.71 \pm .02$ during the four years of the experiment.

3. Students whose scores are in the upper range in both tests have higher scores in the old type test. Students whose scores are low in both tests have lower scores in the old type test. This is shown in the correlation table and in the value of sigma which is approximately twice as great for the old type test distribution of scores as for the new type test.

4. It is satisfactory to shift students from one class to another at the same hour only once during a semester. The end of the first six weeks' period is a suitable time for this shift.

5. As measured by the June tests in analytics, it is found that students who are sectioned into three classes at an hour on the basis of high, middle, or low ability, have made greater progress than have students in the control sections. In the old type test the gain is 7.3 percent in favor of the experimental group. In the new type test this gain is 5 percent. If the results of the two tests are combined the gain is 6.3 percent in favor of the experimental group.

These percentages are figured on the basis of perfect test scores.

6. If the *average* test score on the combined tests is used as a basis of comparison rather than the *perfect* score the 6.3 percent gain mentioned above becomes 12.5 percent of the average score.

7. The gain of the experimental sections over the control sections seems to be fairly uniform at all ability levels. There is no reason to believe that sectioning on the basis of ability is more advantageous for the upper quartile than it is for the lower quartile, as earlier experiments often claimed.

8. For the sections where the division is made into two ability groups, high and low, the average gain is neither uniform nor large in magnitude. It is 4 percent on the old type test and less than 1 percent on the new type test. As the experiment was conducted, the evidence of the advantage of dividing classes into two groups at an hour is not conclusive.

9. The effects of classification of students on the basis of ability seem to be practically independent of the instructors teaching the course. A change of instructors each year has changed neither the direction nor the magnitude of the gains.

10. The majority of the instructional staff is in favor of continuing the method of sectioning in analytics.

11. The students' expression of opinion shows that a majority of that group favors the method. The reaction of students in the low sections is much better than it is commonly understood to be.

12. There is some evidence from the study of the percentages of grades given in the following course, differential calculus, that students who have been in the experimental sections in analytics are less apt to make *E* grades, and have more chance to make *D* and *C* grades than have students of all sections. The number of *A* and *B* grades combined are almost the same for each group.

13. This study of all second semester classes in analytic geometry at the University of Illinois for a four-year period offers reliable evidence as to the effects of sectioning college students on the basis of previous grades in mathematics. How far one may generalize from this experiment as to the effects of sectioning in other courses depends upon finding a population of which the group treated may be considered a random sample.

The Problem of the Poorly Prepared Mathematics Teacher in Our Secondary Schools[1]

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So large a percentage of college freshmen who have taken all the mathematics the high school has to offer are poorly prepared to continue mathematics that one is moved to ask: "Why not demand well prepared mathematics teachers, if the subject is important enough to be taught?"

A Liability, Not an Asset to the System

There are three reasons why the incompetent mathematics teacher is a liability and not an asset to the school system: (1) Mathematics is basic to science and engineering. As a fundamental subject it should be taught only by well prepared teachers. (2) The average student has difficulty with mathematics and, even with a competent teacher, masters the material only through intensive study. He does not get it with a poorly prepared teacher but rather gets discouraged and keeps away from mathematics. (3) The technical reasoning processes of the subject make it necessary to have capable mathematics teachers, not incompetent ones.

An Instance Cited of the Work of the Poorly Prepared Mathematics Teacher

One of the excuses for teaching geometry in the high school is that it develops the reasoning power of the student. Consider the reasoning power that can be instilled into the youthful mind by a teacher such as this woman (located in a large class A high school): In a six weeks test she asked the following question: "If the perimeter of an equilateral triangle is given, is its area determined?" Several of the students recalled how she had only a short time before emphasized the theorem stating that the area of a triangle is fixed if its base and altitude are given. Without much reason-

[1] This article is intended to be a constructive criticism of a condition existing in many of our secondary schools.

ing these boys and girls answered the question: "No." They received full credit for their answer. One of the superior students answered the question by writing: "Yes. The sides are fixed, hence there is only one triangle possible, so that its area is determined." His reply was marked wrong, yet he had reasoned entirely correctly and had given the correct answer. Imagine the respect the superior student had for his geometry teacher's reasoning!

Too often even in the better class of schools, mathematics teachers emphasize the memorizing of formulas, theorems, and exercises rather than the more useful method of reasoning. The teacher mentioned above is a product of poor teaching. She has memorized, not logically reasoned out her geometry. More pitiful is the fact that she is passing her illogical reasoning on to others, while being paid to teach geometry—a course in the curriculum because it is supposed to teach boys and girls to think and reason. Consider what a liability this teacher is to the school system of which she is a part. Why are there so many incapable mathematics teachers?

Combination Teaching Positions

A common way to handle the mathematics situation, especially in the small school system, is to hire the athletic coach or debate teacher (two customary combinations) and give him mathematics classes to fill out his schedule.

Recently a certain college placement bureau had failed to place two mathematics majors—one with a bachelor's degree, the other with a master's degree. The same placement bureau, through no fault of its own but rather through the fault of a misguided school board and superintendent, was able to place an athlete in a combination coaching-mathematics position. The fellow that was placed had had no college mathematics whatever. The board told him that it would be necessary for him to go to summer school to obtain three hours credit in college algebra. What a preparation!

Other combinations as ridiculous as this one are effected time and time again. I know of a teacher of manual training and woodwork who has classes in high school algebra. In order to become more capable, the fellow is taking trigonometry and wants to learn but he isn't even aware of the principles of arithmetic and elementary algebra. How he teaches algebra in the high school with such a weak foundation is more than I can see.

The Difficulty

The preparation of the athletic director or debate teacher is quite thorough in the schools where we find poorly prepared mathematics teachers. Such can not be otherwise since the community usually demands a winning football or basketball squad and a convincing debate team. This is as it should be, except that the athletic director or debate teacher is (by his contract) required to teach mathematics on the side. He of course is not to blame. Even though he is not prepared to teach mathematics, he can in one summer's study obtain a smattering of mathematics, so that he can keep one or two jumps ahead of his students. This arrangement satisfies entirely too many school boards and superintendents. Seemingly they fail to realize that mathematics is largely a reasoning process which can not be acquired in one short summer of college work. They also fail to realize or admit the importance of mathematics in the curriculum of the school, as is evidenced by such combinations.

As a result of the combinations so often effected, the teachers who are really prepared to teach mathematics must teach subjects which they too are unprepared to handle. This condition weakens the whole educational system and should not exist.

A Remedy Offered

In the small school system where the teachers must teach a combination of subjects, combinations of allied subjects should always be made. Usually the college mathematics student is also a student of physics, chemistry, or some other natural science, so that the natural sciences and mathematics combine well. Often students proficient in mathematics are also good in music. The music-mathematics combination could be used more often for satisfactory results. Even certain languages (Latin and Greek especially) and mathematics merge fairly well. The athletic enthusiast is very seldom good in mathematics. This explains why a very common combination so often results disasterously.

"The right teacher in the right place" increases efficiency not only in the mathematics department but in the entire school system and also solves the problem of the poorly prepared mathematics teacher.

The Social Qualities of Mathematics*

By J. T. Rorer
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Mathematics as it should be taught in high schools is both useful and necessary as a preparation for acquiring other useful knowledge and as a helpful daily tool in most life situations. Arithmetic problems need but brief explanations or comment. They are conspicuously social; they are commercial, industrial, economic, governmental, and scientific. In today's parlance, mathematics is a core-subject.

Though its field may be condensed to the electron or expanded to the cosmos, mathematics humbles itself to clarify, systematize, and perhaps measure nearly all human affairs. Its problem material contains budgets, balance sheets, and manifold percentage applications, such as interest, stocks, bonds, taxes, insurance, and annuities. Graphs depict vividly all types of statistics. These curves even share with the "Theory of Probability," the role of prophesy. The simple equation is a simple sentence, whether the language be dead or alive; systems of equations yield compound sentences; the checking of roots illustrates complex sentences, including the subjunctive mood and other afflictions of the grammarians. How can chemistry and physics be studied without formulas, equations, and graphs? After a lesson in a modern classroom on $s=\frac{1}{2}gt^2$, it takes little imagination to see the apple falling on Sir Isaac's crown, a phenomenon that brought forth the law of gravitation, itself to be succeeded by Einstein's mystic equations understood only by super-mathematicians.

Music is a form of mathematical symbolism with its eighth, quarter, half, and whole notes. The lovely melody which delights our ear is rhythmic vibration mathematically designed and controlled to avoid the horrors of continuous disonance and the torment of barbaric noise.

Even the educators must use our terms to clarify and ornament their rhetoric. A single address at the Washington meeting of the N.E.A. contained "integrate" at least ten times. Personality was

* A paper presented at the Ninth Annual Conference on Secondary Education, Teachers College, Temple University, Philadelphia, October 27, 1933.

integrated, the behavior child was integrated, and the inferiority complex was doubly integrated. One always wishes that one knew more mathematics when these professionals wade into the calculus! Perhaps it may be postulated that some of our beloved colleagues, teachers of other subjects,—wish that they knew more mathematics when the Department of Educational Research talks of medians and modes, coefficients of correlations, probable errors, etc. The National Committee, more than ten years ago, advocated the teaching of simple statistics in our schools, and urged that elective opportunity be given high school pupils to understand the true meaning of function, limit, differentiate, integrate, etc. In many schools, sad to say, these important suggestions have not been carried out.

History is one of our chief social studies. There is opportunity to teach much vital history in the mathematics classrooms. What part has counting played in the up-lift of mankind? What economic part has algebra played? What advantage has the geometry of Copernicus to that of Ptolemy? We owe much to the Greeks especially in geometry. Would their methods of counting satisfy us today? What did Napier and Briggs do for us? The great Newton, Leibnitz, Descartes, and a host of other discoverers have done more for us than Julius Caesar and Napoleon Bonaparte.

The reading of current literature requires far more mathematics than was necessary twenty or more years ago. A recent front page news item in "The Philadelphia Public Ledger," describing some discovery in atomic structure, spoke of a metric distance 10 to the minus 24th power. What does it mean? Even the juvenile magazines, the Sunday puzzle pages, and the air-craft columns of the evening papers contain considerable mathematics. The young readers bring clippings to the classrooms sometimes with the request, "Please explain this"; sometimes proudly with the statement, "I was able to solve this," or "I understand this because of our work in mathematics."

Appreciation is a key work of modern pedagogy. We use it, too. We look at nature, art, architecture, great bridges, wonderful machines, with increased appreciation because of our knowledge of geometric forms, of the laws of their being, and of their action. The snow flake, the rainbow, the "Morning Glory," the spider web, the honey-comb, all increase in beauty and grandeur when viewed by intelligent rather than savage eyes.

Mathematics today is more than counting; more than the science of number, quantity and form. These essential ideas are to mathematics what oil, pigment, and canvas are to a great painting! Mathematics is a spirit, an art, a science, impossible to define. These are some few of the qualities, attitudes, motives, habits, and skills:

1. Clearness of concept. Our symbols must be understood to be properly used.

2. Economy of thought and expression. We continually abbreviate, and as we advance, our symbols are more and more inclusive.

3. Preciseness and accuracy. These qualities are properly synonymous with mathematics. They need only one comment. That is, we have greatly overworked them. Human beings, while ever striving for accuracy, always will make mistakes. We cannot expect 100% accuracy, nor do we. Yet the present pupil is so woefully inaccurate that we must teach him to find his error and to correct it. This is an important part of our job, and is a much more useful viewpoint than that of slaying him because he makes mistakes.

4. The habit of analysis—of separating the essential from the non-essential; the hypothesis from the conclusion; the possible from the impossible, under the conditions at hand.

5. The habit of logical reasoning—from simple premises step by step to a definite conclusion. To attain this end, geometry is the best instrument known because it deals with such simple cases, far simpler than other studies. Associated with this is the appreciation of what we mean by a "proof." A step in advance is research, and the joy of discovery—the frequent reward of the tenth grader. Under the guidance of the skillful teacher the habit of logical thinking produces the reflective mind.

6. Beauty, admiration, and love of thought processes and the thought-cosmos. This is the highest goal of the teacher. Mathematics yields to no other field the potentiality to approach this goal as a limit! This was in the mind of Plato, when he said, "God is continually geometrizing." Jeans has been inspired with the same thought when he declared in 1930, "The Great Architect of the Universe now begins to appear as a pure mathematician." Some feel these statements so extravagant as to be almost devoid of meaning. Those who are skeptical should read, "Poetry and Mathemat-

ics," by Scott Buchanan, or Bell's "Queen of the Sciences," or Keyser's "Mathematical Philosophy."

7. The call of the teacher of mathematics today, his great objective, if you will, is far away from answer-grinding. It is rather to do his part with the means at hand, to train youth to be happy, thoughtful, appreciative, right minded, useful men and women, who have at least a glimpse of "mathematics as the music of reasoning" rather than an ambition merely to rival the mechanical precision of the Comptometer.

Advanced Mathematics in the Secondary School*

By Ralph Dennison Beetle
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In response to a widespread desire expressed by both colleges and schools that the courses in advanced mathematics be revised, the College Board has appointed a committee to act in the matter. Since this committee will endeavor to produce results satisfactory to the majority of the constituency which it represents, it behooves us, both individually and collectively, to formulate well-considered answers to the question: *What is the most desirable content of the secondary school courses in advanced mathematics?*

I have no cut-and-dried answer to propound and defend. I am more in the position of setting before you some of my own convictions and some of my own perplexities in the hope that the ensuing discussion will aid me in arriving at more definite conclusions.

In view of the many objectives of education in these days, and the many effective means of attaining these objectives, we can hardly expect to obtain any unique solution which can rigorously be proved the best possible. It is rather a question of effecting a compromise between conflicting desires and purposes with the ultimate answer depending essentially upon the relative weights assigned to the considerations accepted as pertinent.

There is an important difference between the so-called advanced mathematics and the elementary mathematics which precedes it. While from a broader point of view all the courses in question must be regarded as very elementary in character, the world as a whole finds logical plane geometry so abstruse that it considers almost all further developments as material for the specialist. In advanced mathematics, therefore, we have our first instance of courses voluntarily elected because of enjoyment of the subject or recognition of its value for the individual.

Mathematics, regarded only a few decades ago as fundamental in any program for collegiate education, has been steadily losing in prestige, and is now definitely on the defensive, almost with its

* Read on October 28, 1933, at a meeting of the Association of Teachers of Mathematics in New England.

back against the wall. Like Latin, it would be crowded into oblivion were it not recognized as indispensable in an age in which our daily lives are dominated by the marvels of science and engineering.

With nothing beyond plane geometry required for admission to most colleges, and with virtually no requirement of mathematics in many colleges, there is no logical reason why the colleges should have much to say about the content and viewpoint of the secondary courses in advanced mathematics. Of course, due consideration must be given to the students who expect to continue their study of mathematics in college, but such consideration should not be, and need not be, at the expense of those for whom the courses in advanced mathematics furnish the climax of their mathematical training as part of their formal education.

We hear often of the difficulties imposed upon the schools because of the domination of the colleges. We are assured that, given a free hand, the schools can develop programs better designed to meet the needs of their pupils. In recent years, new methods of admission have relieved the tension somewhat and there are indications that it will not be long before the schools will be dictating to the colleges.

Unquestionably the secondary schools have many responsibilities other than that of preparing pupils for college entrance. I do not dispute the validity of the assertion by David Eugene Smith that "The ideal plan is to give the pupil the best that can be given in each of the school years, so that if he leaves at the end of any given year he will have the best all-round view of the subject, up to that point, that we are able to offer."* I do, however, question the accuracy of some supposedly logical deductions from this fundamental principle and others of a similar nature.

I cannot accept it as self-evident that there is inevitable conflict between the best interests of those who are continuing their studies and those who are not. Moreover, the element of time is so important in mathematical development that we are led to absurdities if we carry our applications of the principles too far.

If you were an Englishman privileged to spend but a single day of your whole life in America, is it not conceivable that you would gain more profit and enjoyment by staying right in New York City than by flying to San Francisco and back with a Hawks or a Doolittle?

* The Mathematics Teacher, March, 1921, p. 124.

I still believe in developing the elementary mathematics naturally and logically at a pace slow enough to permit a normal individual to gain both a reasonable proficiency in the essential technique and also an intelligent grasp of the underlying theory. If a pupil is forced to leave school, or if he abandons mathematics to graze in the greener pastures of other subjects, he may go through life completely ignorant of the important knowledge which was just around the corner, but he will be better off with a clear-cut remembrance of the few topics he was able to master than with a hazy distorted outlook upon a wider range.

I do not mean that I am satisfied with the present courses as outlined in the requirements of the College Board. I am well aware that they contain many curious groupings of topics, as well as illogical details of inclusion and omission, and it is high time that they were revised. But I hesitate to replace the present inflexible system by another equally inflexible. I believe that only a system with a maximum of flexibility will meet all the objections which can be raised to the present system.

I am puzzled as to the attitude of teachers in this connection. While some teachers are advancing ideas which call for very great flexibility, others (or perhaps the same ones) are utilizing to the utmost every iota of inflexibility in the present requirements.

For example, the requirements in elementary algebra state that "the pupil should be required to construct the graph of such an expression as x^2-2x+3." Seven or eight years ago a committee of examiners asked for the graph of x^3-x, innocently regarding it as an expression fully comparable to x^2-2x+3. In fact, is it not a little simpler? One coefficient has been reduced from 2 to 1; the constant term has been changed from 3 to 0. Will not these simplifications more than offset an increase of the exponent from 2 to 3?

Not at all! Hundreds of teachers raised a storm of protest and insistence that limitation to curves of the second degree was clearly implied. Is it not then equally implied that the coefficient of x cannot exceed 2 or the constant term 3? I know nothing about the intent of the committee which originally framed the requirements, but I regard it as absurd to claim that a pupil trained to plot the graph of x^2 cannot be expected to produce the graph of x^3. At least, it makes no sense unless we presuppose curious modes of instruction.

Even though it introduces elements of difficulty in the setting

of satisfactory college board examinations, I should like to see the colleges and the schools agree upon a very flexible program for advanced mathematics which would, on the one hand, include a minimum list of topics regarded as essential by the colleges, but would, on the other hand, permit the schools the greatest possible freedom in the choice of the remaining topics. I believe that this freedom of the schools could be great enough to allow each school ample scope for self-expression. In particular, this procedure should obviate any necessity for differentiation in the treatment of those preparing for college and those who are not.

However, I wish to make it clear that I disapprove of some of the programs likely to be introduced if this freedom becomes an actuality. Some authorities advocate the introduction into current courses of material which normally belongs in more advanced courses. This is recommended as a means of displaying the erudition of the teacher so as to command greater respect from the pupils, and is also excused on the ground that a glimpse, as through a spyglass, into the colorful fields of distant mathematics engenders enthusiasm.

I believe that both of these purposes are better served by lateral enrichments of the current topic, of a nature not naturally included in courses of the immediate future. To be more specific, I am opposed to the introduction into the secondary school of the calculus in any form, of projective geometry, of analytic geometry as such, or of any special stress on the concept of function.

A real understanding of the viewpoint and significance of such topics requires a maturity of thought not ordinary present in the secondary school classroom. If any teacher is fortunate enough to have pupils capable of genuine appreciation of such a topic as the calculus, it is proper that they be given full opportunity to use their extraordinary abilities to the utmost, but algebra, geometry, and trigonometry have phases which will tax to the utmost the mental powers of any freshman in Dartmouth College. Yet we are getting some of the cream from the secondary schools, or else we are being grossly deceived by our officers of admission.

I approve of replacing the present three half-year courses by a single continuous year-course in advanced mathematics, but I prefer to confine the content of the new course to topics from the old courses until a case for new material can clearly be proved on the basis of its merit.

I still believe in developing the elementary mathematics naturally and logically at a pace slow enough to permit a normal individual to gain both a reasonable proficiency in the essential technique and also an intelligent grasp of the underlying theory. If a pupil is forced to leave school, or if he abandons mathematics to graze in the greener pastures of other subjects, he may go through life completely ignorant of the important knowledge which was just around the corner, but he will be better off with a clear-cut remembrance of the few topics he was able to master than with a hazy distorted outlook upon a wider range.

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The precise selection of topics, except for certain basic ones on which all can agree, and the order of presentation of the topics, I should leave as far as possible to the teacher, but I am absolutely opposed to any attempt to present the advanced mathematics as one logical unit bound together by some such unifying principle as the concept of function. I speak here from actual experience in teaching courses so conceived. The result is inevitably to create unnatural orders and extremely artificial viewpoints which confuse the student and annoy any teacher who is not a fanatic on unification and motivation.

Because we regard a polynomial as a simpler type of function than any transcendental function, we are not logically compelled, or even impelled, to discuss the theory of equations before proceeding to trigonometry and logarithms. For some purposes it is convenient to list the members of a class in alphabetical order but we do not feel constrained to ask them to recite in that order, nor do we bother about beginning with the youngest, the shortest, the lightest, or even the prettiest.

When authors omit permutations, mathematical induction, and other topics, simply because they fit awkwardly into the functional scheme of order, and when the same authors postpone the numerical solution of triangles until long after the differentiation of the logarithmic function, the dog is certainly being wagged by his tail, or rather by something that has been mistaken for his tail.

There are, however, other types of unification which have received less publicity, yet appear to have much more merit. One of the mysteries of modern education is the consistency with which otherwise competent and careful teachers disregard in trigonometry fundamental principles previously established in algebra and geometry.

Trigonometry is really nothing but a part of geometry in which the attention is concentrated on certain ratios of line segments. In proving the theorems of trigonometry, it is found convenient to use algebraic methods very freely. Here then is a real chance for natural unity of treatment. How is this splendid opportunity capitalized?

First by ignoring the fundamental principle of algebra that a non-zero number cannot be divided by zero. Textbook writers and teachers alike are determined that no angle shall suffer the disgrace of being without all six trigonometric functions. Actual errors

of statement are occasionally avoided by elaborate explanations or evasions, but it is a rare teacher or student who will frankly admit, without some apology for such astounding destitution, that poor ninety degrees has no tangent, never has had one, and never can have one.

Secondly, too few teachers realize that, since every identity of trigonometry is actually a theorem of geometry, its proof must be established by processes as logically correct as those employed in the ordinary geometry. In plane geometry emphasis is placed on the fact that the truth of a theorem does not imply the truth of its converse. Yet many teachers permit their students to handle identities by procedures which are logically equivalent to claiming that a theorem must be true because the validity of its converse has been demonstrated.

In the light of situations of this sort, some of the extravagant claims made for mathematical instruction as a panacea for all human ills become slightly humorous. I believe in boards of education, departments of education, and even experimental and progressive schools, but I do not trust them to do all my thinking for me. It is human nature to accept leadership from those who gain the limelight through their own competence and enthusiasm for progress, but the teachers of this country are no *Light Brigade* doomed to do and die with no right to reason why.

Occasionally, it seems to me, educators are inclined to promulgate programs utterly impossible of realization under the existing conditions of personnel, time limitations, and so forth, when more modest programs, which take more definite account of actual conditions, are more likely to be successful.

To carry out the suggestions of many state pamphlets, it is not enough that a teacher thoroughly understand his subject and its important applications, and have the faculty for transmitting this understanding to his pupils. He must have in addition the combined abilities of salesman, showman, historian, painless dentist, and jack of all trades. He must convince the pupil—not once, but day by day—of the value, not only of mathematics in general, but also of the individual topic under discussion; he must then render the acquisition of this worthwhile knowledge not only free from pain but actually one of positive enjoyment.

Thus New Jersey says: "*The only tenable position for teachers of mathematics to take*" is that "*presentation must recognize the fact*

*that the subject is interwoven with life, that it has elements that may be identified in countless life situations, and that our aim is to reveal to pupils of differing abilities the broadest meaning of the subject for all human experience."**

The Commonwealth of Massachusetts is more explicit and detailed, but no more modest, in its statement of the aims of instruction in mathematics in the Junior High School Grades.† Since most of you must be familiar with that statement, I will merely summarize the high lights.

The pupil should be led to think of mathematics as a useful tool, an interesting field of knowledge, a most important mode of human thought, and an indispensable aid to the progress of civilization. He should be led to desire to learn his own strengths and weaknesses, and to develop himself through growth in accuracy, responsibility, and good judgment. He should learn to desire knowledge and to seek this knowledge through personal investigation, meanwhile cultivating habits of understanding, analytic reasoning, and breadth of outlook. Finally, properly conducted instruction should foster social attitudes of self-confidence and public-mindedness which will enable the pupils to picture themselves as ultimately successful, through thrift and honesty, in those vocations to which mathematics has been applied in the classroom.

The official statement has eight or nine hundred words. I have reduced it to about 125, keeping most of the big words. I had to leave out some of the ideas; if you wish to know all that mathematics can do for the adolescent boy or girl, you will have to go to the original.

I will make no further comment except to ask two questions. Is there time to teach anything but mathematics in the Junior High Schools of Massachusetts? If these are the modest aims in those grades, what are considered sufficiently ambitious aims for the twelfth grade? Unfortunately, I had no pamphlet to answer these questions for me.

I do not object to these lofty purposes if they can be translated into actualities. It is more important that a student become a good citizen than that he become proficient in mathematical manipulations, but it is not clear that he cannot become both, nor is it clear that his ultimate value as a citizen is jeopardized if he occasionally concentrates upon mathematics as such without being forced

* Syllabus in Ninth Year Algebra, January, 1933, p. 5.

† Bulletin of the Department of Education, 1926, No. 6, p. 4.

simultaneously to consider its bearing upon his own character and upon the destinies of humanity.

Perhaps I err in supposing that there is any high degree of correlation between what is printed in the state bulletins and what actually happens in the classroom, but those of us who deal with the products of secondary school instruction sometimes wonder whether some teachers, in their eagerness to display the value and significance of mathematics, do not forget to teach the mathematics itself.

It is not that I lack faith in mathematics, nor do I underestimate the courage, capacities, and earnestness of its teachers, but I believe that the best philosophies of life and the best evaluations of human experiences come, not from classroom instruction, but from life itself through our contacts with men and with things.

By this time some of you must be thoroughly convinced that my ideas are a heritage from the dark ages of thirty or forty years ago and that I am surprisingly resistant to the influences of the present age of progress. All right, I will now stop and let you tell me so.

Mnemonic Devices—Old and New

Samuel H. Barkan
Abraham Lincoln High School, Brooklyn, N.Y.

Although in the mathematics teaching of to-day we stress reasoning and thought, we find it convenient sometimes to have our pupils memorize facts that are used frequently. To assist our pupils in retaining these facts, most teachers use devices of various sorts. May I mention a few of the most common tricks that I have used with success?

1. In working with radicals, it is well to know by heart the decimal value of the square root of 3. The result, 1.732, has the same figures as the year of Washington's birth. The square root of 2, which is 1.414, does not exactly have an historical analogy, but I usually inject a touch of humor by saying that this figure is easy to remember because it is exactly 39 years before the fall of Constantinople.

2. The value of *pi*, 3.1416, can be remembered by using the mnemonic, "Yes, I have a number." The number of letters in each word is the same as the corresponding digits in 3.1416.

3. In working with the formulas for arithmetic progression, the letters involved spell out the word *LANDS*. In working with geometric progressions, the letters used in the formulas spell out the word *SNARL*.

4. Students sometimes forget the three things that should be mentioned in describing the nature of the roots of a quadratic equation. The word *ERR* can be used in remembering the words *Equal, Real, Rational.*

5. While learning the meaning of fractional exponents, pupils sometimes experience difficulty in changing x to the $\frac{3}{4}$ power to radical form. I have found it helpful to tell pupils that the numerator is on top and is therefore "in power" whereas the denominator is underneath, like the "root" of a tree. Hence the result is the fourth "root" of x to the third "power."

6. Theorems in geometry that involve mean proportionals can be easily remembered by what I call the "slide" method. For instance, the theorem about the secant and tangent can be thoroughly impressed by having the pupils say, "The secant is to the

tangent as the tangent is to the external segment" and at the same time by having the pupil SLIDE his pointer along these lines without lifting the pointer from the blackboard. In this group of theorems the pupil will find that he will slide over the mean proportional TWICE.

7. Humorous situations in the classroom sometimes serve as mnemonic devices. Pupils have told me that they have remembered certain things because of some funny incident. For example, one pupil could not remember which was the numerator and which the denominator of a fraction. I told him that the word "denominator" began with the letter D as in the word "down," and that the word "numerator" started with the letter N as in the word NUP. This was pure nonsense but it had the desired effect.

8. When learning the binomial theorem, it is well to point out to the pupils that the coefficients of the successive expansions can be arranged in the following scheme:

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array} \quad \text{and so on.}$$

I have used these and similar devices with some success and would be glad to hear of any other stunts that have been tried by readers of the Mathematics Teacher.

On To Cleveland

George Berkeley, Bishop of Cloyne

Born at Dysert, Kilkenny, Ireland, March 12, 1685
Died at Oxford, January 14, 1753

It was in the year 1734 that Bishop Berkeley made his famous attack upon the doctrine of fluxions, which was the starting-point of all philosophical discussion of the new mathematics in England during the eighteenth century.* Bishop Berkeley is better known as a philosopher and as a theologian than as a mathematician. Yet the publication of his essay *The Analyst* has been characterized as "the most spectacular event of the century in the history of British mathematics.** And present day students will be interested in this two hundred year old attack on the methods of the calculus.

Berkeley's father was an English customs officer stationed in Ireland, and Berkeley was connected with that country for most of his life. He went to Trinity College, Dublin, in 1700 remaining there as fellow and as tutor after he took his degree. He became dean of Dromore in 1722, dean of Derry in 1724, and Bishop of Cloyne in 1734. These appointments seem to have carried suitable stipends but seem to have permitted long absences. In fact while Berkeley held the first of them, he was also acting as lecturer in Hebrew at Dublin. His interest in Ireland is shown by his publication in 1735–37 of *The Querist* in which he discussed social and economic conditions in that country.

On leaving Trinity College in 1712, Berkeley went to London where Swift presented him at court. From 1713 to 1720 Berkeley travelled in France and Italy acting as chaplain or as tutor to a succession of wealthy patrons.

While in Dublin, Berkeley had written two short mathematical tracts (1707). Two years later he published *An Essay toward a new Theory of Vision.* In 1721, after the speculative fiasco of the South Sea Bubble, he produced *An Essay toward preventing the Ruin of Great Britain.* At about the same time he proposed the founding of a "College or Seminary in some convenient part . . .

* Florian Cajori, *A History of the Conceptions of Limits and Fluxions in Great Britain from Newton to Woodhouse.* Chicago, 1919 p. 2. This discussion is largely based on this material and on the excerpts from *The Analyst* which appear in the *Source Book in Mathematics* edited by David Eugene Smith, New York, 1929.

** Cajori, p. 57

where the English youth of our plantations may be educated in such sort as to supply the Churches with pastors of good morals and good learning." The churches were naturally those of the Church of England, and the college was to be located in Bermuda. Berkeley devoted his attention to this project for several years in the course of which he received the appointments mentioned above and during which he also received a considerable legacy from Hester Vanhomrigh, the Vanessa of Swift's poem, who, after a quarrel with Swift, changed her will and left half of her estate to Berkeley whom she had met but once.

In 1728, Dean Berkeley had the promise from Parliament of the money needed for his college and sailed for America settling at Newport where he remained for three years. While there, he decided to locate the college on the mainland of North America, but the promised funds were not forthcoming and he returned to England. At the time of his departure, he wrote to Samuel Johnson (who was then at Yale but who later became president of King's College in New York) to inquire whether part of his library would be acceptable to Yale. Upwards of a thousand books were shipped to the college and today the Yale library proudly lists the contents of the various "Berkeley boxes." At the same time he gave his Rhode Island property to Yale to provide a graduate scholarship. It is quite fitting that one of the buildings of the Berkeley Divinity School in New Haven should be patterned after Berkeley's home in Newport.

Mention has been made of two short mathematical tracts which appeared before Berkeley left Trinity College, Dublin. Between that time and Berkeley's return from America, he seems to have done nothing with the subject, but he undoubtedly followed the popular interest in its new developments. It should be remembered that at that time the fluxions of Sir Isaac Newton and the differential calculus of Leibniz were tools which were being used with little regard to mathematical rigor. Cajori says of this that "No writers, unless we except Newton (1704) and Ditton, dispense with the use of infinitely small quantities. The dropping of such quantities from an equation was usually permitted without scruple.

"What an opportunity did this medley of untenable philosophical doctrine present to a close reasoner and skilful debater like Berkeley!"*

* Ibid. p. 56.

The title of Berkeley's essay gives the purpose in detail: *The Analyst: or, a Discourse addressed to an Infidel Mathematician. Wherein it is examined whether the Object, Principles, and Inferences of the Modern Analysis are more distinctly conceived, or more evidently deduced, than religious Mysteries and Points of Faith.* The "Infidel Mathematician" is supposed to be Halley. What Berkeley really thought about fluxions is a matter of debate. Sir William Rowan Hamilton, the inventor of quaternions, said "On the whole, I think that Berkeley persuaded himself that he was in earnest against Fluxions." And De Morgan had no doubt that Berkeley knew that they were sound.

It is difficult to select illustrative passages from the *Analyst* for Berkeley was one of the most skilled debateιs of his day and the essay is full of quotable sections, each referring to the author's chief premise. The following paragraph illustrates this:

All these points, I say, are supposed and believed by certain rigorous exactors of evidence in religion, men who pretend to believe no farther than they can see. That men who have been conversant only about clear points should with difficulty admit obscure ones might not seem altogether unaccountable. But he who can digest a second or third fluxion, a second or third difference, need not, methinks, be squeamish about any point in divinity.

He bases much of his discussion on the following lemma:

If, with a view to demonstrate any proposition, a certain point is supposed, by virtue of which certain other points are attained; and such supposed point be itself afterwards destroyed or rejected by a contrary supposition: in that case, all the other points attained thereby, and consequent thereupon, must also be destroyed and rejected, so as from thenceforth to be no more supposed or applied in the demonstration.

He says "This is so plain as to need no proof." He then illustrates the point by the case of the fluxion of x^n. If the x has an increment o, then x^n becomes $(x+o)^n$ and its increment is $nox^{n-1}+\dfrac{nn-n}{2}oox^{n-2}+\cdots$. The ratio of these increments is then found to be as 1 is to $nx^{n-1}+\dfrac{nn-n}{2}ox^{n-2}+\cdots$. From this point let us follow Berkeley's treatment.

Let now the increments vanish, and their last proportion will be 1 to nx^{-1}. But it should seem that this reasoning is not fair or conclusive. For when it is said, let the increments vanish, *i.e.*, let the increments be nothing, or let there be no increments, the former supposition that the increments were something, or that there were increments, is destroyed, and yet a consequence of that supposition, *i.e.*, an ex-

pression got by virtue thereof, is retained. Which, by the foregoing lemma, is a false way of reasoning. . . .

The great author of the method of fluxions felt this difficulty, and therefore he gave in to those nice abstractions and geometrical metaphysics without which he saw nothing could be done on the received principles: and what in the way of demonstration he hath done with them the reader will judge. It must, indeed, be acknowledged that he used fluxions, like the scaffold of a building, as things to be laid aside or got rid of as soon as finite lines were found proportional to them. But then these finite exponents are found by help of fluxions. Whatever is got by such exponents and proportions must be ascribed to fluxions: which must therefore be previously understood. And what are these fluxions? The velocities of evanescent increments. And what are these same evanescent increments? They are neither finite quantities, nor quanties infinitely small, nor yet nothing. May we not call them the ghosts of departed quantities?

The work concludes with a number of queries, two of which are of especial interest to students.

Qu. 16. Whether certain maxims do not pass current among analysts which are shocking to good sense? And whether the common assumption that a finite quantity divided by nothing is infinite be not of this number?
Qu. 54. Whether the same things which are now done by infinites may not be done by finite quantities? And whether this would not be a great relief to the imaginations and understandings of mathematical men?

Replies to the *Analyst* were made by James Jurin, a physician who had been a student of Newton's, and by John Walton who was professor of mathematics in Dublin. Jurin comments on Berkeley's work in these terms*

Your attack, I surmise, is really not so much in the interest of Christianity, as to demonstrate your superiority as a reasoner, by showing Newton and Barrow, two of the greatest mathematicians, less clear and just than you are.

Later, having met Berkeley's arguments point for point, he says:

Having now . . . driven you entirely out of your intrenchments. . . . I should sally out and attack you in your own. . . . But as they seem rather designed for shew, than use . . . to dazzle the imagination . . . (they) will likewise disappear like *the Ghost of a departed quantity*, if you exorcise them with a few words out of the first section of the *Principia*.

Berkeley replied with *a Defence of Free-Thinking in Mathematics* (1735). To this both Jurin and Walton wrote rejoinders, Jurin's

* ibid, p. 65. Jurin's reply is dated April 10, 1724, thus putting the publication of the *Analyst* in late March or early April for England at that date was still using the Julian calendar and reckoned March 25 as the beginning of the new year.

having the title "*The Minute Mathematician: or, The Free-Thinker no Just-Thinker*. In commenting on Berkeley's lemma, he says:

Let us imagine yourself and me to be debating this matter, in an open field, . . . a sudden violent rain falls . . . we are all wet to the skin . . . it clears up . . . you en-

deavor to persuade me I am not wet. The shower, you say, is vanished and gone, and consequently your . . . wetness . . . must have vanished with it.

. . . .

I meet with nothing in my way but *the Ghosts of departed* difficulties and objections.

Berkeley's reply to Walton called forth a second edition of Walton's first comment on Berkeley. At this point Berkeley withdrew from the controversy which was to continue with other participants for several years, but in 1744 he wrote this footnote to a discussion of absolute space and absolute motion:*

Our judgment in these matters is not to be overborne by a presumed evidence of mathematical notions and reasonings since it is plain the mathematicians of this age embrace obscure notions, and uncertain opinions, and are puzzled about them, contradicting each other and disputing like other men: witness their doctrine of Fluxions about which, within ten years, I have seen published about twenty tracts and dissertations, whose authors being utterly at variance, and inconsistent with each other, instruct by-standers what to think of their pretentions to evidence.

VERA SANFORD

* Ibid. p. 179.

Fifteenth Annual Meeting of the National Council of Teachers of Mathematics

PROGRAM

HOTEL CLEVELAND, CLEVELAND, OHIO, FEBRUARY 23 AND 24, 1934

General Theme: The Present Crisis in Secondary Mathematics

Friday, February 23, 1934

10:00 A.M.—Business Meeting of the Board of Directors.

12:30 P.M.—Director's Luncheon

2:00 P.M.—*First General Meeting*

Topic: The Future of Geometry in the High School

Speakers:

1. Professor Ralph Beatley, Harvard University, on behalf the geometry committees of the National Council.
2. Mr. Rolland R. Smith.
3. Discussion.

Friday Evening

6:00 P.M.—Informal Supper of the Delegates representing the Affiliated Organizations, followed by Reports of the Delegates.

8:00 P.M.—*Second General Meeting.*
Address of Welcome. (Speaker to be announced.)
A Mathematical Symposium, on the General Theme of the Meeting. Presented by members of the National Council, arranged and directed by Miss Mary A. Potter and assistants.

Saturday

9:00 A.M.—Annual Business Meeting.

10:00 A.M.—*Third General Meeting.*
Topic: The Problems of Ability Grouping and of Differentiated Curricula.

1. *Some Aspects of the Administrative Phases of this Problem.* Dr. C. N. Stokes, Temple University, Philadelphia.
2. *Remedial Work in Arithmetic, A Challenge and a Warning Signal.* Miss Genevieve L. Skehan, Whitney School, No. 17, Rochester, N. Y.
3. *The Problem of Individual Differences and a Suggested Attempt for its Solutions.* Miss Clara D. Murphy, Evanston Township High School, Evanston, Ill.
4. *An Experiment in Teaching Graphs.* (An illustrated demonstration.) Mrs. Wilimina E. Pitcher, Rawlings Junior High School, Cleveland, Ohio.

2:00 P.M.—*Fourth General Meeting.*
Topic: What Can We Do to Meet the Challenge of the Present Situation in Secondary Mathematics?
Speakers:

1. Professor W. D. Reeve, Teachers College, Columbia University.
2. Discussion. A group of four-minute addresses. (Speakers to be announced.)
3. Professor J. O. Hassler, The University of Oklahoma, on behalf of the Policy Committee.

Saturday Evening

6:00 P.M.—*Banquet and Final Meeting.*
Greetings from Guests of Honor.
Address: Mathematics and Music. Professor Carl A. Garabedian, St. Stephen's College, New York

NEWS NOTES

The Mathematics Section of The Central Association of Science and Mathematics Teachers held its annual meeting at the Congress Hotel in Chicago on Friday December 1, 1933, with Mr. Maurice L. Hartung of the University High School of Madison, Wisconsin presiding. The following program was given: Appointment of Nominating Committee; "Achievement Testing in Secondary Mathematics," H. T. Lundholm, The Blake School, Minneapolis, Minnesota; "Dimensionality," Prof. E. P. Lane, University of Chicago; "Geometry's Tribute to Tradition," Dr. Elizabeth B. Cowley, Pittsburgh, Pennsylvania; General Discussion; Election of Officers.

The following note from one of our subscribers is heartening:

"Herewith find $2 for renewing my subscription to The Mathematics Teacher. It grows better from year to year and I do not want to miss a single copy. I have files from the time when it was published by the Middle Atlantic States and Maryland Association."

The Mathematics Section of the New York State Teachers Association, Western Zone, held its annual meeting at Buffalo, N. Y. on Nov. 3, 1933.

The following program was given:

10:00—Address: "The Use of Materials from the History of Mathematics in High School Work," Dr. Vera Sanford, Oneonta, N. Y.

11:00—Discussion groups:

"Problems in the teaching of Elementary Algebra and Plane Geometry," led by Miss Christiana Hathaway, Head of Mathematics Department, Niagara Falls Senior High School.

"Problems in the teaching of the advanced mathematics courses of the high school," led by Joseph F. Phillippi, Professor of Mathematics, State Teachers College at Buffalo.

1:45—Business Meeting.

2:00—Report of an investigation: Can Dull Pupils Learn? Prof. Raleigh Schorling, Head of Mathematics Department, University High School, University of Michigan.

The Ninth Annual Conference on Secondary Education was held on October 27 and 28, 1933, at Teachers College, Temple University. The Conference Topic was "New Orientations in Education for Participation in a Changing Social Order." For Group 10 (Mathematics) the Chairman was Dr. C. Newton Stokes, Assistant Professor of Education, Teachers College, Temple University.

The program was as follows:

Friday, 4:15–6:00 P.M.: Scheduled Participants:

Science: Robert E. Kunzig, Olney High School, Philadelphia.

Guidance: Robert Briggs, Counselor, West Philadelphia High School.

Home Economics: Marjorie Sims, Drexel Institute, Philadelphia.

Fine Arts: Ralph Yonkers, Head of Art Dept., West Philadelphia High School.

Commercial: Thomas Milne, Head of Commercial Dept., Upper Darby High School, Upper Darby, Pa.

Health: Charles O. Roeser, Shaw Junior High School, Philadelphia.

Social Studies: Leon S. Drumheller, Overbrook High School, Philadelphia.

Music: Wilbert Hitchner, Supervisor of Music, Wilmington, Del.

Industrial Arts: Charles Riley, Radnor High School, Wayne, Pa.

Mathematics: Vera Sanford, Oneonta, New York; Fletcher Durell, Belle Plain, N. J.; Clarence Garbrick, Simon Gratz High School, Philadelphia; Jonathan Rorer, Head of Mathematics Dept., William Penn High School, Philadelphia.

Guiding Questions: Does the mathematics program of the secondary school develop those powers of understanding and analyzing relations of quantity and of space which are necessary to a better appreciation of the progress of civilization and a better understanding of life and of the universe about us, and develop those habits of thinking which will make these powers effective in the life of the individual by:

1. Developing dispositions and abilities to (a) maintain health and physical fitness? (b) use leisure time in right ways? (c) sustain successfully certain definite social relationships such as civic, domestic, community and the like? (d) engage in exploratory-vocational activities?

2. Acquiring fruitful knowledge which is (a) preparatory to acquiring other knowledge? (b) directly functional in developing dispositions and in discovering and developing abilities? (c) useful in the control of situations of everyday life?

3. Developing attitudes, interests, motives, ideals and appreciations?

4. Developing definite mental techniques in memory, imagination, judgment and reasoning?

5. Acquiring right habits and useful skills?

Saturday, 9:15–11:00 A.M.: Scheduled Participants: Vera Sanford Fletcher Durell, Clarence Garbrick, Paul Whiteley, Mrs. Charlotte Klewans, Lela Lynam, Elizabeth Wood, Percival M. Fogg, Margaret Groff, Marian Lukens, Pauline Vanden Beemt.

Guiding Questions: In what aspects has the mathematics program of the secondary school failed:

1. To draw upon the problems of life sufficiently to give the individual a better understanding of life and thus to enable him to make better adaptations to his environment?

2. To develop those abilities needed to discover or create the useful life when the individual is using his own faculties?

3. To develop the proper attitudes toward the quantitative aspects of life?

4. To develop the dispositions and abilities to make correct interpretations of number experiences?

In the light of modern theory:

1. What reconstructions and reorganizations of the mathematics program are necessary in order that the individual may be given the experiences which will result in a functional residue?

2. Could this reconstruction and reorganization be best accomplished through (a) the co-operation of two or more departments on a common unit of the curriculum? (b) a realignment of present divisions so that the mathematics department would assume certain responsibilities now vested in other departments? (c) a reconstruction and reorganization of the mathematics curriculum in the light of life needs, irrespective of the effects upon the offerings of other departments?

The Fall Meeting of the Connecticut Valley Section of the Association of Teachers of Mathematics in New England was held at Trinity College in Hartford, Connecticut, on Nov. 4, 1933.

The following program was given:

Morning Session: 10:15—Social Gathering in Boardman Hall; 10:30 "Modern Concepts of Parallelism," Dr. Alfred K. Mitchell, Trinity College, Hartford; 11:00—Presentation and discussion of preliminary report by the Committee on Geometry. N. A. Jackson, Mount Hermon, Chairman; F. L. Mockler, Holyoke; Dr. B. H. Brown, Dartmouth; 12:45—Business Meeting; 1:00—Luncheon.

Afternoon Session: 2:15—Presentation and discussion of preliminary report by the Committee on Advanced Mathematics. Dr. D. D. Leib, Conn. College, Chairman; C. G. Ross, Mount Hermon; Beatrice Neal, Bulkeley High School.

The Officers of the Connecticut Valley Section are: Professor H. M. Dadourian, Trinity College, Hartford, President; Carroll G. Ross, Mount Hermon School, Vice-President; Arthur D. Platt, Mount Hermon School, Secretary; Miss Mary Noyes, High School, Bristol, Treasurer; Walter Blaisdell, Nathan Hale Junior High School, New Britain, Director; Miss Dorothy S. Wheeler, Bulkeley High School, Hartford, Director.

Past Presidents are: Harry B. Marsh, '20; Percy F. Smith, '21; M. M. S. Moriarty, '22; Eleanor C. Doak, '23; Joe G. Estill, '24; Joshua I. Tracey, '25; Lyon L. Norton, '26; John W. Young, '27; Rolland R. Smith, '28; Harriet R. Cobb, '29; Melvin J. Cook, '30; Bancroft H. Brown, '31; Dorothy S. Wheeler, '32.

The Second Meeting of the Men's Mathematics Club of Chicago, was held on November 17, 1933.

The following program was given:

1. "Mathematics and The Changing Curriculum," Edwin S. Lide, University of Chicago.
2. "What We Are Doing at New Trier High School," Wm. A. Snyder, New Trier High School.

The first meeting of the Cleveland Mathematics Club for the year 1933–34 was held in the Auditorium of the Board of Education Building, Tuesday, November 14, at 4 o'clock.

Superintendent Lake addressed the Club, his subject being "Modern Methods of Financing Education."

The Association of Mathematics Teachers of New Jersey held its forty-ninth regular meeting at Convention Hall in Atlantic City N. J. on Nov. 11, 1933.

The following program was given:

General Theme: "The Place and Teaching of Graphical Methods in Ninth Grade Mathematics."

Panel Speakers:

1. "Graphing the Linear Equation of Two Unknowns," Miss Theresa A. Featherston, Jefferson High School, Elizabeth.
2. "Solving Simultaneous Linear Equations with Graphs," Miss Ida E. Housman, Demarest High School, Hoboken.
3. "The Graphical Solution of the Formula," Dr. Frank J. McMackin, Dickinson High School, Jersey City.
4. " Expressing the Tangent Graphically," Mr. Ferdinand Kertes, High School, Perth Amboy.
5. "Using the Graph in Problem Solving," Mr. Michael McGreal, West Side High School, Newark.

Discussion led by Professor Richard Morris, Rutgers University, New Brunswick.

The Officers of the Society are: Miss Amanda Loughren, Thomas Jefferson High School, Elizabeth, N. J., President; Dr. Frank J. McMackin, Dickinson High School, Jersey City, N. J.,

Vice-President; Andrew S. Hegeman, Central High School, Newark, N. J., Secretary-Treasurer.

Council Members are: The Officers and Dean Luther P. Eisenhart, Prof. Charles O. Gunther, Prof. Richard Morris, Virgil S. Mallory, Harrison E. Webb, Harold I. Palmer, Roscoe P. Conkling, Ferdinand Kertes.

The Mathematics Section of the Maryland State Teachers' Association, with a membership of one hundred and fifty members, elected the following officers for 1932–1933.

Chairman: Miss Nanette Roche, Supervisor of Mathematics in the Junior High Schools of Baltimore, 3 East 25th St. Baltimore, Md.

Secretary: Mrs. Evelyn Nicholson Spurgis, Towson High School, Towson, Baltimore County, Md.

Treasurer: Miss Agnes Herbert, Clifton Park Junior High School, Baltimore, Md.

Program of the two meetings, 1932–1933: Fall meeting, October 21, 1933 at the Baltimore City College, Baltimore, Md. Organization, election of officers, and informal address by Dr. Francis Murneghan, Professor of Mathematics at Johns Hopkins University. Spring meeting, April 22, 1933, Cumberland, Md. Afternoon: Round Table Discussion: Leader, The late Miss Annabel Lee White, Vice Principal of Forest Park Senior High School, Baltimore, Md. Topic. The Maximum Efficiency in Teaching Junior and Senior High School Mathematics.

The following papers were read:

"The Teaching of Intuitive Geometry in the Junior High Schools," Miss Nanette Roche; "The Teaching of Plane Geometry," Miss Leola Dixon; "An Experiment in Teaching Mathematics at Catonsville High School," Miss Edna Schwartz; "Motivation in Teaching Mathematics," Mrs. Evelyn Nicholson Spurgin; "A Review of Arithmetic," Miss Naomi Crowl; "Problem Solving," Miss Caroline Mulliken. Evening: Banquet at the Algonquin Hotel; Speaker, Dr. John R. Clark; Topic, "What are the Fundamentals?"

The Winthrop College Branch of the National Council of Teachers of Mathematics (Rock Hill, S. C.) held its regular meeting at four o'clock November 7, 1933 in Kinard Hall.

After a short business meeting the program was continued as follows:

"Mathematical contest directed by Lillian Henderson; "Mystical Mathematics," Mary Louise White; "Mathematical Clubs in High Schools," (1) Arousing of Interest, Louise Weill—(2) Organization, Elizabeth Harmon—(3) Programs, Birdie Jones.

The Winthrop College Branch of the National Council of Teachers of Mathematics held its regular meeting at four o'clock in Johnson Hall Tuesday, November 21, 1933.

The roll was answered with a mathematical joke, after which Mary Wells told the story of "Merlin and Viviane." Margaret Salley read a poem, "The Changeable Changeless Naught." Interesting "Believe It Or Not" items were given by Helen Dowdle.

A social hour was enjoyed after the program was presented.

Elizabeth Harmon
Local Editor

NEW BOOKS

The Administration of Mathematics in Secondary Schools. By Ernest Breslich. University of Chicago Press, 1933. VI+407 pp. $3.00.

This is the third of a series of three volumes which Professor Breslich has written. All three volumes are devoted to the Teaching of mathematics in secondary schools. The first volume, the *Technique of Teaching Secondary School Mathematics*, deals with problems arising in the choice and use of general Teaching procedure and the second, *Problems in Teaching Secondary School Mathematics*, is concerned with specific teaching problems. In his third volume Professor Breslich classifies administrative problems as they relate (1) to the direction and supervision of a department and (2) to the curriculum. The first part is intended to assist the supervisor in organizing the department of mathematics into a unified and cooperative group for the purpose of improving instruction. He discusses such supervisory functions as visitation of teachers, individual and departmental conferences, and the training of teachers. Several chapters are given over to the development of objectives, the formulation of a testing and measurement program, provisions for individual differences in ability, and remedial teaching. The second part relating to curriculum problems gives prominence to how content material should be selected for teaching purposes, the organization and distribution of the various parts of mathematics like arithmetic, geometry, and algebra, how these parts are unified, their organization into teaching units, and the relation of mathematics to modern educational trends.

Professor Breslich's long experience not only as a teacher, but as a supervisor qualify him to speak authoritatively upon any subject related to the teaching of mathematics and this book will be of great value to experienced teachers of mathematics who feel the need of help as heads of departments of mathematics or as supervisors of mathematics in secondary schools.

Not only does Professor Breslich present his own views on many topics, but through a very thorough perusal of the literature of the field and by means of extensive bibliographies he has furnished teachers an opportunity for reading that will help them better to solve their problems and to improve their thinking

It is to be regretted that a book of this type is necessarily placed at such a high price that in these times it will not be availabe to a large number of teachers. This is in the last analysis due to the fault of the teachers themselves who often do not buy such books in large quantities even though the price be fairly low.

The New Day Junior Mathematics, Book III. By Vevia Blair. Charles E. Merril Co., New York and Chicago, 1933, XII+430. pp. $1.24

This book is the last objective evidence of Vevia Blair's untiring efforts to contribute to the reorganization and teaching of mathematics in the secondary school. An artistic teacher she was endeavouring always to make mathe-

matics live as a dynamic force in the lives of the pupils whom she taught. This book gives positive evidence of her attempt to integrate not only arithmetic, algebra, geometry, and trigonometry during the junior high school period but also to show how mathematics contributes to the other great branches of learning like science and the arts. In this sense the book is the outcome of many years of actual classroom experience at the Horace Mann School for Girls at Teachers College, Columbia University.

An attempt is made to make the function idea the organizing principle throughout the book and the pupil is led to think in terms of the dependence of one quantity or another.

In addition to practical and interesting applications of algebra and the more informal aspects of geometry the book contains a chapter on geometry where an approach to demonstration is the objective.

Ample material for pupils of varying ability is offered so that the teacher can make a wise selection. All progressive teachers will be interested to see this book.

First Days with Numbers. By Clifford B. Upton. American Book Co. 1933, 160 pp., $0.40.

This interesting little book represents something new for young pupils and is intended to serve as an easy and natural introduction to number facts and relationships. As a result, the common activities of children, their play and games are used as a basis for the number stories and problems.

The vocabulary has been carefully selected, the illustrations are well chosen, the number facts seem to be arranged carefully and test exercises are included with keyed references to remedial exercises.

College Algebra. By Joseph B. Rosenbach and Edwin A. Whitman. Ginn and Co. 1933, XI+394 pp. $2.00.

This book is intended to afford a new presentation of algebra of first year students in colleges and technical schools who offer either two or three semesters of high school algebra for entrance. It is written so as to bereadily adaptable for either long or short courses.

The book has grown out of an experiment carried on by the authors in their day and evening classes at the Carnegie Institute of Technology. Elementary algebra is fully reviewed in the earlier pages and in the later chapters rather full treatment is given of such topics as complex numbers; theory of equations; logarithms; interest and annuities; permutations, combinations, and probability; determinants; partial fractions; and infinite series.

Teachers who are interested in a book which is the outgrowth of actual classroom experience will want to see this volume.

Differential Equations. By Max Morris and Orley E. Brown. Prentice Hall, Inc., 1933, XII+404 pp. $2.50.

This book has been prepared in such a way as to offer the student an opportunity to secure a generous amount of mechanical manipulation in integrating the various standard types of differential equations while at the same time affording him as wide contact as possible with the more exacting and searching aspects of mathematics.

The exercises are carefully chosen, are numerous and provide a desirable type of motivation for subsequent study and thought. Applications to geometry, mechanics, and the like are chosen so as enable the student to make the proper use of his work.

Descriptive Geometry. By F. H. Cherry. The Macmillan Co., 1933, XI+127 pp., $2.00.

This book is an introduction to engineering graphics, prepared under the editorship of E. R. Hedrick. The text was written in an attempt to overcome the present antipathy of students that has grown up around descriptive geometry. The authors had tried to select from all of the traditional propositions only those that his experience in the classroom has convinced him are the most important and practical from the engineer's standpoint.

The book is planned to cover the work of one semester. The fundamental ideas and concepts are presented in a more or less ideal spiral plan so as to familiarize the student with their use.

Handbook of Mathematical Tables and Formulas. By R. S. Burington. Handbook Publishers, Inc., Sandusky, Ohio, 1933, 251 pages, $2. Special price to students and instructors, $1.25.

This handbook is intended as a required reference book for all students of mathematical courses, and also workers in such fields as statistics, engineering and the like. It begins with the more elementary theorems and formulas of high school mathematics and extends to the material necessary to meet the needs of the college student of mathematics or those of persons using mathematics as an instrument of his profession.

The use of a separate volume of formulas and tables is often advisable and people who find a need for such a book will find this volume of interest.

Diagnostic and Remedial Teaching in First Year Algebra. By Guy E. Buckingham, Northwestern University Contributions to Education, 1933, XI+136 pages.

In this book the author presents the results of an experimental attempt to facilitate the development of technique in the teaching of algebra comparable to previous analysis of the difficulties in the fundamental processes in arithmetic and the subsequent remedial devices that have been provided.

The experimental work was carried on by the author in the New Trier Township High School at Winnetka, Illinois. Supplementary studies were carried on by others the results of which are also reparted in the study.

Those who cooperated in the study feel that the procedures which they employed were of distinct value in overcoming pupil difficulties in algebra and the materials assembled in this volume are intended to be of help to teachers in the field.

Second Year Algebra. By David A Rothrock and Martha Anne Whitacre. Charles Scribner's Sons, 1933, XIV+272 pages, $1.12.

This book is intended to afford a one semester course in intermediate algebra although it includes a comprehensive review of elementary algebra and enough additional work of a more advanced type to meet the needs of the more gifted pupils. This means that the teacher should use his judgment as to the wisest selection of content material.

Detailed explanation and many illustrative type problems are given in order that the book may be as nearly self teaching as possible. An attempt is made not only to improve the pupil's understanding of the underlying theory of algebra but also to help to give him a better appreciation of the importance of mathematics in everyday life.

Differential Equations. By Abraham Cohen. D. C. Heath and Co., 1933, VII+33 pages, $2.40.

This is an elementary treatise on differential equations and is a revision of an original edition. The revision consists in revising certain exercises where that seemed desirable and in general increasing the number of exercises especially where interesting and important applications of both ordinary and partial differential equations to the Physical Sciences could be made. The author has also made some changes at the suggestions of others who have been interested in the improvement of the original edition.

College teachers of mathematics will be interested to see this new text.

Recent Developments in the Teaching of Geometry. By J. Shibli, x+252 pp., State College, Penn. Price $2.25.

This is one of those rather exceptional books on teaching that has genuine merit. It shows a rather unusual study of the literature of the subject and it gives a helpful summary of the ideas expressed by recent writers in this field, both in monographs and in textbooks. The author begins with a brief sketch of the history of the teaching of geometry. His purpose is evidently to lead the teacher of today to look upon mathematics as a moving rather than a static subject. It is quite safe to say that no one is prepared to consider the factors leading the present impetus in the improvement in the teaching of geometry (Chapter II of the work) without the background which some introduction of this kind gives. It is of course impossible to cover the historical ground with any thoroughness in the space allowed—for example with respect to the geometry used by the cathedral builders beginning particularly in the 13th century—but for the purposes of this book the survey is satisfactory. In the second chapter the influences leading to recent developments are considered, not only in this country but in Europe. Here again the treatment is introductory rather than comprehensive, but it is scholarly so far as it goes and it serves the purpose of letting us look over our national walls and getting at least a slight glimpse—too slight it must be confessed—at what our colleagues in other countries are doing.

The modern problem in the teaching of geometry begins in Chapter III, "Intuitive Geometry," a subject which the mathematician might say is not geometry at all, but which he would agree is or should be a desirable introduction to the logical treatment of the subject. Chapter V, "Foundations of Geometry" is, for the teacher who is beginning his work, one of the most valuable of all; for it gives him, in a brief summary, some idea of the real mathematical basis upon which he must build. It is worth a great deal to know what Lobachevski and others of the 19th century did for geometry, opening up an entirely new realm. To appreciate the significance of a postulate is to make healthy progress in the modern teaching of the subject.

Chapter VI gives an idea of what has been done in the last few years to establish a minimum list of propositions which may serve to reveal to the pupil what geometry has to give him in the way of pleasure and of a mental equipment. It should show the teacher the futility of attempting to "cover plane geometry," and establish for him the essentials upon which he can, through "originals," build an interesting and valuable structure. This is made the more clear in Chapters IX and X.

As would be expected, at least by those who have given much thought to the subject, the inferences which might be drawn from the answers to questionnaires and from transient statistics would not be very conclusive, but the statistics serve a purpose when rightly used. That one book has 120 "propositions" while another has 60 would not prove that the former is twice as usable in practice, and similarly as to corollaries, postulates, exercises, and pages; indeed, quite the reverse conclusion might be a better one. Dr. Shibli wisely refrains from drawing conclusions in any such way, and it is to be hoped that the readers will profit by his reticence.

David Eugene Smith
Teachers College
Columbia University

Problem of the Men, the Monkey, and the Cocoanuts

Five men discovered a pile of cocoanuts. By agreement, one of these men, on the next morning after the discovery, came to the place, divided the pile of cocoanuts into five equal piles, but discovered that he had one over. This extra cocoanut he gave to the monkey, took one of the five equal piles and departed. On the next morning, the second man came, threw the four remaining piles into one and divided this pile into five equal piles, but, like the first man, found that he had one over, which he gave to the monkey. He took one of the five piles and departed. This process continued until the fifth morning, there being always one cocoanut over, which was given to the monkey. They all met on the sixth morning and divided the remainder into five equal piles, but had none over. How many cocoanuts were there in the beginning?

—Contributed by C. V. Dunn, Daisetta, Texas